# Population Projection 

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## Three Elements of Projection

- One or more numbers describing the state of the population at any given time $t$.
- A dynamic expressing how this state changes over time.
- One or more parameters describing the rate of change.
- The first two elements are structural, the third is quantitative.


## Simplest Case: Malthusian Projection

- State is defined as total population size at time $t$, denoted $P(t)$.
- Dynamic is defined by $P(t)=P(0) \exp \{r t\}$ where $r$ denotes the exponential growth rate.
- The parameter is the exponential growth rate $r$.


## Formulation in Discrete Time and Continuous Time

- Malthusian projection is readily formulated in continuous time, but projections of disaggregated population are generally formulated in discrete time.
- The discrete time formulation of Malthusian projection is $P_{i+1}=P_{i} \exp \{r n\}$, where time increments are $n$ units long and $\mathrm{P}_{\mathrm{i}}$ denotes population size at time ni.


## Projection Cycles

- Projection begins with the state at some initial time $t$.
- The dynamic is applied to the initial state to generate a 1st projected state for time $t+n$.
- The dynamic is applied to the projected state for time $t+n$ to generate a projected state for time $\mathrm{t}+2 \mathrm{n}$.
- These projection cycles are continued as long as desired.


## The Three Steps of a Single Component Projection Cycle

- Project survivors of the initial population at the end of the period
- Project births to the population during the period
- Project survivors of births during the period


## Age Groups and Time Periods

- Discrete projection is simplified if the length of age groups and the length of time periods are identical.
- Most population projections are carried out using five year age groups and time periods; single year time periods and age groups are sometimes used.


## Projection Mechanics: Input

- Initial age distribution, 5 year groups followed by open-ended age interval
- Life table (abridged) expressing mortality risks; use ${ }_{n} L_{x}$ column to compute survivorship ratios
- Age-specific birth rates, 5 year age groups
- Sex ratio at birth


## Projection Mechanics: Single Projection Cycle - 1

- Simplest Case: Females Only
- "Survive" initial age distribution, 0-4 to 5-9, 5-9 to 10-14, and so on; open-ended interval requires special handling
- Average initial and projected numbers in ages 15-19, ..., 45-49


## Projection Mechanics: Single Projection Cycle - 2

- Apply age-specific birth rates to generate total numbers of births during time period
- Apply sex ratio factor to get total female births from total births
- Apply life table survivorship ratio to determine number of survivors of births


## Repeat Projection Cycles

- These steps may be repeated for as may cycles as desired; 20 cycles will project the population age distribution forward 100 years
- Of course we may change the projection parameters at each projection cycle to allow for changing fertility and mortality


## Questions? <br> Comments? <br> Discussion?

