#### **Population Projection**

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#### Three Elements of Projection

- One or more numbers describing the **state** of the population at any given time *t*.
- A **dynamic** expressing how this state changes over time.
- One or more **parameters** describing the rate of change.
- The first two elements are *structural*, the third is *quantitative*.

### Simplest Case: Malthusian Projection

- *State* is defined as total population size at time *t*, denoted *P*(*t*).
- Dynamic is defined by P(t) = P(0)exp{rt} where r denotes the exponential growth rate.
- The *parameter* is the exponential growth rate *r*.

# Formulation in Discrete Time and Continuous Time

- Malthusian projection is readily formulated in continuous time, but projections of disaggregated population are generally formulated in discrete time.
- The discrete time formulation of Malthusian projection is P<sub>i+1</sub> = P<sub>i</sub> exp{rn}, where time increments are n units long and P<sub>i</sub> denotes population size at time ni.

### **Projection Cycles**

- Projection begins with the state at some initial time *t*.
- The dynamic is applied to the initial state to generate a 1st projected state for time t+n.
- The dynamic is applied to the projected state for time *t*+*n* to generate a projected state for time t+2n.
- These **projection cycles** are continued as long as desired.

The Three Steps of a Single Component Projection Cycle

- Project **survivors** of the **initial population** at the end of the period
- Project **births** to the population during the period
- Project **survivors** of births during the period

# Age Groups and Time Periods

- Discrete projection is simplified if the length of age groups and the length of time periods are identical.
- Most population projections are carried out using five year age groups and time periods; single year time periods and age groups are sometimes used.

## **Projection Mechanics: Input**

- Initial age distribution, 5 year groups followed by open-ended age interval
- Life table (abridged) expressing mortality risks; use <sub>n</sub>L<sub>x</sub> column to compute survivorship ratios
- Age-specific birth rates, 5 year age groups
- Sex ratio at birth

Projection Mechanics: Single Projection Cycle - 1

- Simplest Case: Females Only
- "Survive" initial age distribution, 0-4 to 5-9, 5-9 to 10-14, and so on; open-ended interval requires special handling
- Average initial and projected numbers in ages 15-19, ..., 45-49

Projection Mechanics: Single Projection Cycle - 2

- Apply age-specific birth rates to generate total numbers of births during time period
- Apply sex ratio factor to get total *female* births from total births
- Apply life table survivorship ratio to determine number of survivors of births

### **Repeat Projection Cycles**

- These steps may be repeated for as may cycles as desired; 20 cycles will project the population age distribution forward 100 years
- Of course we may change the projection parameters at each projection cycle to allow for changing fertility and mortality

Questions? Comments? Discussion?