

## Untilting Age Distributions: A Transformation for Graphical Analysis

*This article presents a new approach to the plotting of age distribution data. "Untilting" is a way of transforming data that vary systematically from very high to very low values so as to show local variation more clearly. The article derives an untilting transformation from the formal structure of age distributions. The transformation turns out to be closely related to two familiar demographic techniques, reverse-survival estimation of births and birth rates, and comparison of observed with stable age distributions. The ideas are illustrated by application to age distributions from the 1979 and 1989 censuses of Vietnam.*

by Griffith Feeney

The past several decades have seen the development of increasingly sophisticated techniques for estimating levels and trends of fertility and mortality. Valuable as these developments are in particular applications, they do not eliminate the need for understanding the quality and characteristics of the data to which they are applied. This understanding must usually be acquired through a careful scrutiny of the data, using relatively elementary tools. There is a danger that naive enthusiasm for elaborate formalism will result in the neglect of simpler and more traditional, but essential, preliminaries.

*Griffith Feeney is a demographer at the East-West Population Institute and Affiliate Graduate Faculty in Population Studies at the University of Hawaii, Manoa.*

Age distributions are perhaps the most fundamental of all demographic data. Accurate age distributions contain a great deal of population history, and an appreciation of the errors present in age data is fundamental to the study of many populations, both historical and contemporary.

The first step in analyzing an age distribution is to plot it and scrutinize the result against whatever background is provided by context. When numbers of persons decline very rapidly with increasing age, however, changes from one age group to the next tend to be obscured by the overall downward sweep of the plotted points. We can hope to improve a plot that is sharply tilted in this way by "untilting," that is, by transforming the plotted values so as to make the smaller ones roughly equal in magnitude to the larger ones. The vertical axis may then be rescaled to display fluctuations more clearly.

These ideas are not new in the demographic literature, going back at least to Carrier and Farrag (1959), but they seem to have been largely lost in recent years. The authoritative United Nations *Manual X* (UN, DIESA, 1983) does not discuss them, for example, nor does its predecessor, *Manual IV* (UN, DESA, 1967). The weighty Shryock and Seigel (1973) cites the Carrier-Farrag paper in the chapter on age composition, but the text is silent. Interestingly enough, the graphical procedure of plotting against oblique axes used by Carrier and Farrag is identical to the general approach to untilting described by Tukey (1977:154-156).

This article presents a new approach to the plotting of age distribution data based on an untilting transformation derived from the formal structure of age distributions. Plotting is regarded here not merely as a means for displaying known results, but also as an important tool for data analysis. This view has been developed over the past several decades by a number of writers, including Tukey (1977), Tufte (1983), Wainer (1984), and Cleveland (1985). The untilting procedure turns out to be closely related to two familiar demographic techniques, reverse-survival estimation of fertility and comparison of observed with stable age distributions.

### ■ The problem illustrated

Figure 1 plots the age distribution of the total population of Vietnam at the census taken on 1 October 1979. The shape is characteristic of rapidly growing populations, with large numbers of persons in the younger age groups and small numbers in the older age groups. The downward sweep of the plotted points with increasing age is so strong that departures of particular age groups from the general trend are difficult to make out.

Because this pattern is typical of developing countries, conventional plots of their age distributions tend to look very much the same. Discrimination being a prime purpose of plotting, the plots are unsuccessful. The naive conclusion is that we may as well not have bothered. The correct conclusion is that we need more effective plotting techniques.

Several comments on Figure 1 are in order. First, we are concerned here with plots as tools for data

analysis. Although the population pyramid so popular in demographic methods texts and elsewhere may have its uses, data analysis is not one of them. We want to make plots quickly and with a minimum of effort, and the histogram format of the population pyramid is hopelessly inefficient in this respect.

Equally important, comparisons of plots are best made by overlaying plots drawn on separate pieces of tracing paper. If we want to compare the male and female age distributions, in particular, we want them on separate pages, not side by side. Such is the rationale for the general format of Figure 1.

Two other details are worth noting. We follow the usual convention of plotting the number in a given age interval above the mid-point of the interval. Following Cleveland (1985:31), we put axes and scales on all four sides of the plot and use two scales for each axis when it is useful to do so. Thus the lower

horizontal scale in Figure 1 shows age, whereas the upper scale shows year of birth. The vertical scale on the left is number of persons, while the scale on the right is proportion of total population.

### ■ The untilting transformation defined

Age distributions may exhibit proportionately small numbers of persons at older ages for two reasons. First, older persons have had longer exposure to mortality; hence fewer of them will have survived. Second, older persons were born longer ago, and in a growing population there will be fewer persons born in the past than more recently. These two factors completely determine the shape of the age distribution in a population closed to migration, and they are usually the primary influences in the overall shape of the age distribution even in the presence of substantial migration.

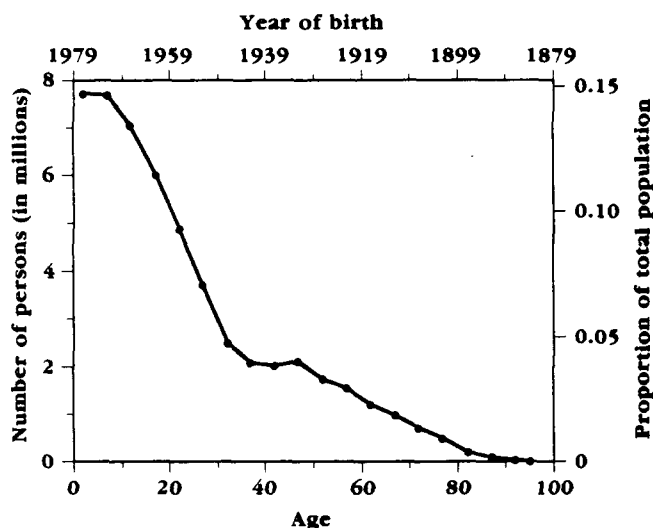


Figure 1. The age distribution of Vietnam, total population, 1979

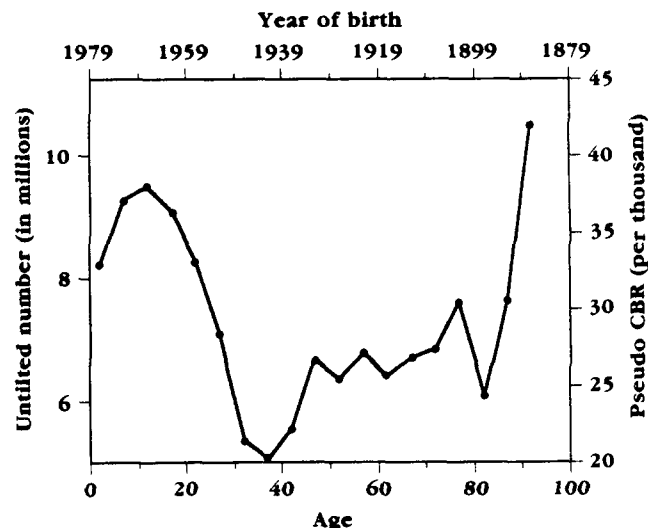


Figure 2. The untilted age distribution of Vietnam, total population, 1979

The tilting effect of mortality on an age distribution may be removed by dividing the number of persons in each five-year age group by the corresponding cohort survivorship proportion, taken as usual to be  ${}_5L_x/5l_0$ , from a suitably chosen life table. Taking the life table radix  $l_0$  to be one, we simplify this to  ${}_5L_x/5$ . The tilting effect of population increase may be removed by multiplying the number of persons in the age group  $(x, x+5)$  by  $e^{rx}$ , where  $r$  is an approximate average annual growth rate for the 80 years prior to the date of the age distribution.

Our first untilting transformation is defined by applying both of these operations. Thus from given numbers  ${}_5N_x$  for five-year age groups we compute

$$\frac{{}_5N_x e^{rx}}{{}_5L_x/5} \quad (1)$$

where  $r$  denotes the growth rate and  ${}_5L_x$  the number of person-years lived in the interval  $(x, x+5)$ .

Although we should utilize whatever knowledge we have of the population to choose sensible values for  $r$  and the  ${}_5L_x$ , it is neither necessary nor appropriate to conduct elaborate investigations.

Our purpose here is simply to untilt the plot, and a good initial guess will often accomplish this. If it does not, we simply adjust the values and try again. It should of course be remembered that the growth rate and mortality levels are likely to have been changing, so that there may be no strictly correct choice.

To untilt the Vietnamese age distribution in Figure 1, we take  $r$  to be the growth rate between the censuses of 1979 and 1989, 0.0210, and apply the  ${}_5L_x$  values from the

Coale-Demeny West Level 19 model life table (Coale and Demeny 1983), with female and male tables averaged, using a sex ratio at birth of 106. The resulting untilted values, calculated from formula (1), are shown in Figure 2, with the vertical scale to the left.

The untilted values for the oldest ages now have roughly the same magnitude as the values for the youngest ages. This makes variation in numbers for the older age groups, invisible in Figure 1 because of their relatively small numbers, as visible as the variation in numbers for the younger age groups. It also makes it possible to "blow up" the vertical scale, rendering all variations more readily apparent.

Figure 2 shows a smooth progression from one age group to the next through age 35, with an initial rise followed by a steep drop; a clearly defined hollow centered on the age group 35-39, corresponding to the birth cohort of 1940-44; minor irregularities between the ages of 40 and 80 and a sharp dip at the age group 80-84; and sharply increasing values for the oldest age groups.

### ■ Untilting and reverse-survival estimation

The untilted age distribution values from formula (1) are absolute numbers, whence comparisons of two or more untilted distributions confound fluctuations with population size. Dividing (1) by  $5Pe^{-2.5r}$  and rearranging terms gives

$$\frac{{}_5N_x/({}_5L_x/5)}{5Pe^{-r(x+2.5)}} \quad (2)$$

where  $P$  denotes the total number of persons in all age groups.

Formula (2) gives relative numbers, removing the effect of total population size. More importantly, it shows that the untilted values, suitably scaled, approximate crude birth rates. The numerators are reverse-survived numbers of births in five-year time periods, and the denominators approximate the corresponding number of person-years lived.

The numerators will be exact if the population is closed to migration and experiences constant mortality corresponding to the given  ${}_5L_x$  values. The denominator is necessarily approximate, except in the degenerate case of a stable population, since actual annual growth rates will vary from year to year.

Because the untilted values (2) differ from the untilted values (1) only by the constant factor  $5Pe^{-2.5r}$ , the plots of the two series are identical except for the scale. Figure 2 shows both scales, with the scale for the formula (2) values on the right.

Table 1 presents the data, intermediate calculations, and results. The first two columns give the  ${}_5N_x$  and  ${}_5L_x$  values. Column C1 shows  ${}_5N_x/({}_5L_x/5)$ , i.e., reverse-survived numbers of births for five-year periods prior to the census. Column C2, calculated as the values in column C1 multiplied by  $e^{rx}$ , gives the result of the first untilting transformation (1). The effect, assuming the usual case of  $r > 0$ , is to inflate the reverse-survived numbers of births in earlier periods. The operation is analogous to the discounting of income and expenditure flows in economics, but applied in reverse. Column C3, calculated as the values in column C2 divided by  $5Pe^{-2.5r}$ , shows the values of the second un-

tilting transformation (2). The final column indicates birth cohorts corresponding to the age groups at left.

The identification of the untilted age distribution values (2) with crude birth rates is valuable, but it is essential to remember that the approximations may be very inaccurate. I shall refer to them as "pseudo CBRs" or as "CBR indices," simultaneously calling attention to the identification and warning against its uncritical acceptance.

### ■ Interpreting the untilted age distribution

Taken at face value, the pseudo CBRs in Figure 2 suggest that the crude birth rate in Vietnam rose sharply, from 20 per thousand in the early 1940s to 38 per thousand in the late 1960s, and fell with equal rapidity to 33 per thousand in the late 1970s. The first question to be addressed is the extent to which this pattern is robust against changes in the untilting parameters.

The effect of choosing a larger

value of  $r$  will be to increase the value of  $e^{rx}$  in (1) more at older ages and earlier years than at younger ages and later years. Hence, if a value of  $r$  that yields an approximately level plot is increased, the effect of choosing a higher value of  $r$  will be to tilt the plot upward, holding the leftmost point fixed. Similarly, choosing a lower value of  $r$  will tilt the plot downward. The overall tilt of the plot reflects the choice of  $r$  and must not therefore be interpreted as a characteristic of the given age distribution.

Figure 3 shows the effect of choosing growth rates of 0.016 and 0.026 as compared with the reference value of 0.021 per thousand. This is a wide range, and there is substantial variation in the levels indicated, but the pattern of rise and fall is untouched.

The effect of choosing different mortality levels is slightly different. Given an initial choice that yields an approximately level plot of the untilted values, choosing  ${}_5L_x$  values corresponding to higher mortality means lower values and hence larger untilted values computed from (1). While the values at older ages will tend to increase more, the values for all the age groups increase, so that the effect is both to lift the untilted values up overall and to tilt them upward. Lower mortality levels reverse this effect, lowering the overall level and tilting the plot downward.

Figure 4 shows the effect of choosing various Coale-Demeny "West" mortality schedules. Because the reference choice was a female life expectancy ( $e_0$ ) of 65 years, a higher than expected value, we consider only lower alternatives,  $e_0$  values of 60, 55, and 50 years.

Table 1. Untilting the 1979 age distribution of Vietnam: both sexes

Age (x)	${}_5N_x$	${}_5L_x$	C1	C2	C3	Years
0	7,712	4,678	8,243	8,243	32.9	1975-79
5	7,691	4,600	8,360	9,285	37.1	1970-74
10	7,040	4,571	7,701	9,500	38.0	1965-69
15	6,015	4,537	6,629	9,083	36.3	1960-64
20	4,882	4,487	5,440	8,280	33.1	1955-59
25	3,718	4,427	4,199	7,099	28.4	1950-54
30	2,492	4,360	2,858	5,366	21.4	1945-49
35	2,071	4,281	2,419	5,044	20.2	1940-44
40	2,004	4,184	2,395	5,547	22.2	1935-39
45	2,109	4,057	2,599	6,687	26.7	1930-34
50	1,728	3,883	2,225	6,359	25.4	1925-29
55	1,554	3,643	2,133	6,770	27.1	1920-24
60	1,204	3,310	1,819	6,412	25.6	1915-19
65	979	2,859	1,712	6,704	26.8	1910-14
70	718	2,275	1,578	6,863	27.4	1905-09
75	496	1,574	1,576	7,611	30.4	1900-04
80	200	881	1,135	6,090	24.3	1895-99
85	91	354	1,285	7,660	30.6	1890-94
90	27	85	1,588	10,513	42.0	1885-89
95	9	9	5,000	36,761	146.9	1880-84
100	3	0	—	—	—	—

${}_5N_x$ —number of persons in age interval  $(x, x+5)$ , in thousands.

${}_5L_x$ —life table person years lived in age interval  $(x, x+5)$ .

C1— ${}_5N_x / ({}_5L_x / 5)$ .

C2— $C1e^{rx}$ ,  $r = 0.0210$ .

C3— $C2/5Pe^{-2.5r}$ , where  $P$  denotes total population, 52,742 thousand.

Year—year in which persons aged  $(x, x+5)$  were born.

Sources:  ${}_5N_x$  values from Vietnam, GSO (1983:34, Table 5),  ${}_5L_x$  values from Coale and Demeny (1983).

This again is a broad range, and the resulting shift in magnitudes is again substantial; but the pattern stubbornly persists. It is, in short, extremely robust against changes in the untilting parameters.

Might the rise-and-fall pattern be due to errors in the age distribution? The smoothness of the changes makes this hypothesis implausible. UN *Manual IV* (UN, DESA, 1967:17–22) shows that, although we may encounter age distribution errors of this magnitude, the pattern is entirely different. Other relevant evidence includes the Vietnamese practice of observing an animal-year cycle and the finding that the single-year age distribution from the 1989 census shows little evidence of heaping (Vietnam, GSO, 1990). Unfortunately, the single-year age distribution for the 1979 census is unavailable. Age-distribution errors are undoubtedly present and perhaps even influential in some respects (we shall

see evidence of this below), but they cannot plausibly explain the suggestion of rising and then falling crude birth rates.

We must conclude, therefore, that crude birth rates in Vietnam really did rise and fall according to the general pattern indicated. More work would be required to get a better fix on the magnitudes involved, but the simple operation of untilting the age distribution has provided a significant conclusion and a useful guide to further analysis.

Analysis of the untilted age distributions at the older ages is in general more problematic, and I shall not pursue its deeper aspects, some of which are discussed in Feeney and Hamano (1990). The essential difficulty is that the values are far more influenced by the choice of untilting parameters, as is evident in the righthand portions of Figures 3 and 4.

Two exceptions are worth noting,

however. Short-term fluctuations, such as the regular up-down-up pattern seen at ages over 40 in Figures 3 and 4, are invariant under all changes in the untilting parameters and must therefore reflect either errors in the reported age distribution or changes in fertility or mortality.

The second exception occurs at the extreme old ages. The sharp rise in the pseudo CBR values for the oldest age groups has already been noted, and in fact the plot omits the pseudo CBR for the 95–99 age group, an obviously absurd 143 per thousand. To understand these observations, note first that the reverse-survived numbers of births from the oldest age groups are obtained by dividing the very small numbers in these age groups by even smaller survivorship proportions, rendering the calculation nonrobust against small errors. It might be suggested that the pseudo CBRs for the older age groups be ignored entirely for this reason.

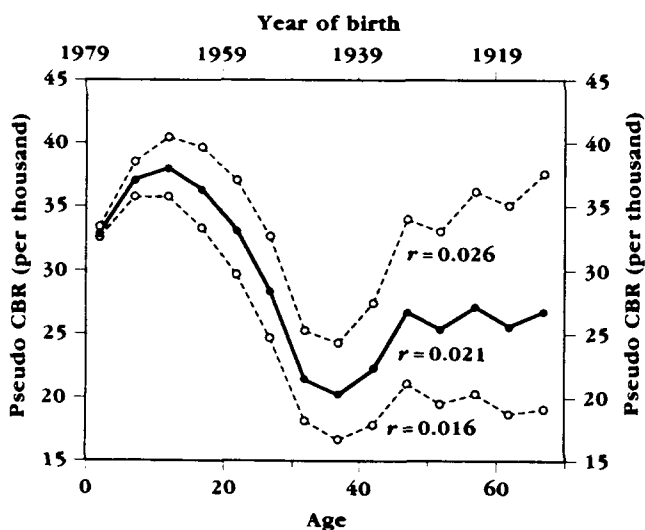


Figure 3. The age distribution untilted, with various growth rates, 1979

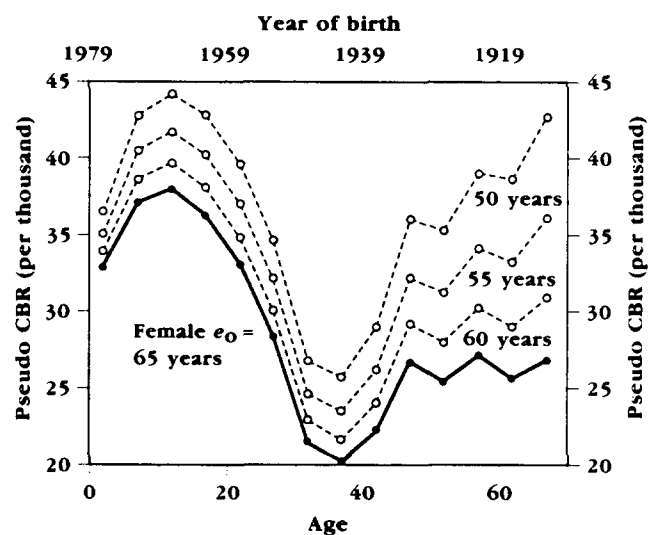


Figure 4. The age distribution untilted, with various mortality levels, 1979

Figure 5. Age-specific sex ratios for Vietnam, 1979

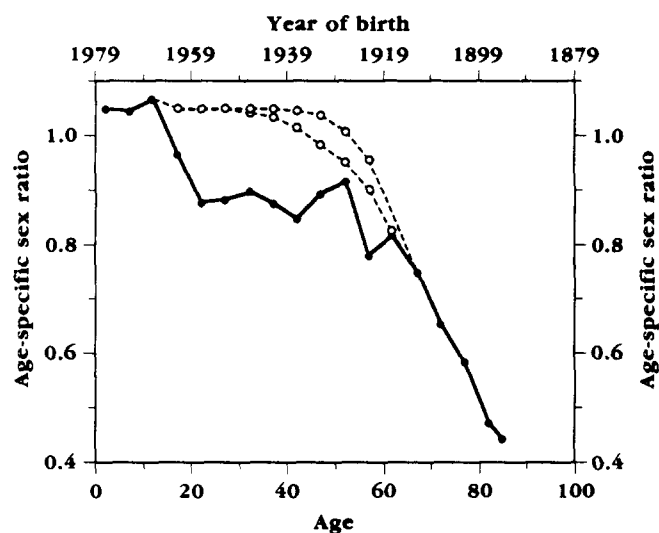
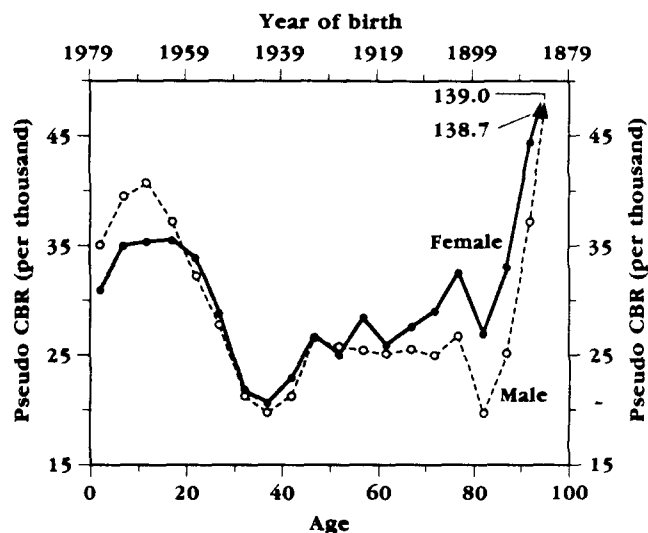


Figure 6. The untilted male and female age distributions of Vietnam, 1979



It turns out, however, that useful conclusions can be drawn from them. Though the values are unstable, they show a persistent empirical tendency to rise with increasing age.

I shall not go into the arguments in detail here, but this almost certainly reflects age exaggeration, which results in greatly excessive numbers in the oldest age groups. A detailed analysis of U.S. data is given in Coale (1990); see also Coale and Kisker (1986).

Having recognized this, we find that sharply increasing pseudo CBRs for the oldest age groups are a useful indicator of probable age exaggeration. The case for age exaggeration in this case is strengthened by the observation that the older cohorts experienced higher mortality than is assumed in the untilting transformation. A life table reflecting this higher mortality would result in even higher pseudo CBRs.

### ■ Sex ratios in Vietnam

Age-specific sex ratios from the 1979 census of Vietnam are plotted in Figure 5 with solid dots from data in Table 2. Notable anomalies are the sharp decline between the 10–14 and the 20–24 age groups, the relatively low values between ages 20 and 60, the pronounced dip in the 55–59 age group, and the leveling off at the oldest ages.

The sex ratios over age 60 are much lower than those in the Coale–Demeny model tables at any relevant mortality level and may represent the “Far Eastern” pattern of mortality identified by Goldman (1980). To provide a plausible reference schedule, we examine life table sex ratios, calculated as the ratio of the male and female  ${}_xL_x$  values multiplied by the sex ratio at birth, taken here as 106 male births per 100 female births.

Life tables for Taiwan over the period 1959–61 and Singapore in 1957, given in Keyfitz and Flieger

(1968:204 and 234, respectively; data in table 2) and intercensal life tables for China, 1953–64 and 1964–82, given in Coale (1984:202), provide useful references. Having plotted all these sex ratios on separate pieces of tracing paper, I placed the Vietnamese plot over the plots for Taiwan and Singapore, then added the upper and lower reference schedules shown in Figure 5 with small circles and dotted lines.

The difference between the reference schedules and the observed schedules represents “missing males” broadly understood. They may be missing because males have been underenumerated relative to females, because males have emigrated in greater numbers than females, or because males have suffered greater mortality than females. The differential mortality explanation stands out as the obvious candidate, given Vietnam’s history of war first with France and then

with the United States; but this explanation is somewhat confounded by the pattern of sex ratios in the 1989 census (data in Table 2). If war deaths were the explanation of the deficit, the deficit would move to the right 10 years between the two censuses. This is not what we observe. The sex ratios in 1989 fall sharply from the 10–14 to the 20–24 age group, just as in Figure 5, and there is no trace in 1989 of the sharp dip at ages 55–59 in Figure 5.

A full analysis of the Vietnamese sex ratios is beyond the scope of this article, but let us consider the question of the "disappearing dip" at ages 55–59 in 1979. Because this dip does not appear at ages 65–69 in 1989, we may be reasonably sure

that it does not reflect a real sex imbalance in the birth cohort. A real sex imbalance occasioned by death or migration in earlier years would persist, and an imbalance at these ages coming into existence during the intercensal period is most improbable.

A possible explanation is that the dip in 1979 is simply due to a typographical error in the census report, but this is quickly dismissed. Not only do the male and female numbers add properly to the total for the age group, but also some three quarters of the provincial age distributions (Vietnam, GSO, 1983:35–74) show the same dip. It is notable that most of the exceptions are southern provinces,

though it is not clear how this observation may be put to use.

A speculation advanced by several members of the General Statistical Office staff was that males of ages 55–59 were overreporting their ages to avoid government-imposed work requirements on working-age men. If this were the case, we should see a dip in the numbers of males, but not of females, in this age group, and a corresponding excess of males in the 60–64 age group.

The observed pattern is readily displayed by untilting the male and female age distributions separately. Figure 6 shows these results. Contrary to expectation, the male age distribution between ages 45 and 80 is relatively smooth. The sex ratio dip appears to be due to a surfeit of females in the age group rather than to a deficit of males.

The up-down-up-down pattern for females between ages 40 and 65 is curious, for it appears to represent age-distribution errors shared neither with males nor with older age groups. Note, however, that it is only the numbers of females in the 55–59 age group that is sharply out of line with the numbers of males.

### ■ Untilting and the stable age distribution

If the given age distribution is stable,

$${}_5N_x = B e^{-\alpha x} ({}_5L_x/5) \quad (3)$$

where  $B$  denotes the number of births in the five-year period preceding the census. We see at once from (1) that the corresponding untilted values will be identical to  $B$ . This suggests that the transformed values may be thought of as representing deviations from a con-

Table 2. Age-specific sex ratios for Vietnam, Taiwan, and Singapore

Age group	Vietnam		Taiwan, 1959–61	Singapore, 1957
	1989	1979		
0–4	1.065	1.048	1.058	1.051
5–9	1.054	1.044	1.059	1.051
10–14	1.062	1.066	1.058	1.050
15–19	0.981	0.965	1.057	1.048
20–24	0.923	0.877	1.052	1.046
25–29	0.907	0.882	1.047	1.046
30–34	0.917	0.896	1.045	1.046
35–39	0.874	0.875	1.041	1.044
40–44	0.869	0.847	1.034	1.042
45–49	0.814	0.893	1.023	1.031
50–54	0.804	0.915	1.004	1.002
55–59	0.882	0.780	0.970	0.956
60–64	0.830	0.815	0.920	0.867
65–69	0.766	0.749	0.848	0.734
70–74	0.678	0.654	0.755	0.598
75–79	0.597	0.585	0.648	0.469
80–84	0.487	0.472	0.524	0.353
85 +	0.434	0.443	0.343	0.217

Sources: Vietnam, 1979, calculated from Vietnam, GSO (1983:34, Table 5). Vietnam, 1989, calculated from Vietnam, GSO (1990, Appendix Table 1.2). Taiwan, 1959–61, and Singapore, 1957, calculated from the male and female  ${}_5L_x$  values in the life tables given in Keyfitz and Flieger (1968:204, 234, respectively), assuming a sex ratio at birth of 1.06 in both cases.

dition of stability.

To obtain a formal result, let us consider the deviation of the observed age distribution from a fitted stable,

$$D_1(x) = {}_5N_x - Be^{-rx}({}_5L_x/5) \quad (4)$$

on the one hand, and the deviations of the untilted age distribution values (1) from  $B$ ,

$$D_2(x) = {}_5N_x e^{rx} / ({}_5L_x/5) - B \quad (5)$$

on the other. Multiplying both sides of the latter identity by  $e^{-rx}({}_5L_x/5)$  gives

$$D_2(x)e^{-rx}({}_5L_x/5) = D_1(x), \quad (6)$$

whence also

$$D_2(x) = D_1(x)e^{rx} / ({}_5L_x/5). \quad (7)$$

Thus we see that the deviations  $D_2(x)$  of the untilted age distribution from the number of births  $B$  equal the deviations  $D_1(x)$  from the fitted stable age distribution transformed by the untilting procedure (1).

If we believe that the true age distribution of the population is stable and have reasonably accurate values for the growth rate  $r$  and the life table survivorships  ${}_5L_x/5$ , the deviations from the fitted stable age distribution may be interpreted as representing errors in the reported age distribution. This is the method of UN *Manual IV* (UN, DESA, 1967:17-22).

## ■ Discussion and conclusion

The variation displayed in the untilted age distribution reflects four factors: the true age distribution of the population; the errors suffered by the reported distribution; the history of fertility, mortality, and

population growth; and the growth rate and survivorship parameters chosen for the untilting transformation. Assigning each factor its proper role is a difficult and sometimes impossible task. This is our natural objective, however, and we aim to carry the analysis of each case as far as circumstances permit. The saving grace is that we are not aiming for definitive conclusions, but rather for various indications to inform subsequent analysis. One or two modest conclusions and a list of sensible questions for further analysis will constitute fair payment for the hour or so of work invested in untilting and plotting an age distribution.

## ACKNOWLEDGMENTS

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